Effects of a Piezo-Actuator on a Finitely Deformed Beam Subjected to General Loading

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The deformation of a beam-column, the upper and lower surfaces of which are bonded in segments with piezo-ceramic liners, is studied for the purpose of obtaining appropriate expressions for the force transferred to the structural member by the piezo-actuator. This concept may be employed for the control of large dynamic deformations of a lattice-type flexible space-structure. The present model, which is based upon a static analysis, accounts for the effects of transverse shear and axial forces in addition to a bending moment on the beam in formulating the governing equilibrium equations. The present model provides more complete expressions for the force transmitted to the structural member than a model reported earlier in literature, in which the shear and axial forces are neglected.

Nomenclature

t_a	= thickness of adhesive
t_p	= thickness of piezo-actuator
t	= thickness of beam column
L	= Length of the segment of the beam column which is lined with a piezo-actuator (Fig. 1)
l	= shortest distance from one end of the deformed beam-column segment to the other (Fig. 2)
G_a	= shear modulus of adhesive
E_p	= Young's modulus of piezo-electric material
E	= Young's modulus of beam column
τ', τ''	= shear stresses on the upper and lower interfaces (Fig. 3)
N_p', N_p''	= Axial forces upon the cross sections of the upper and lower piezo-actuator (Fig. 3)
S_p', S_p''	= shear forces upon the cross section of the upper and lower piezo-actuator (Fig. 3)
σ', σ''	= normal stresses on the upper and lower interfaces (Fig. 3)
M_p', M_p''	= moment upon the cross section of the upper and lower piezo-actuator (Fig. 3)
N, S, M	= axial force, shear force, and moment upon the cross section of the beam column (Fig. 3)
H, V	= horizontal and vertical forces upon the cross section of the beam column (Fig. 2)
$\bar{x}$	$= x/L$
m	$= LM/EI$
h	$= G_aL^2/tt_aE$
h_p	$= G_aL^2/t_at_pE_p$
α	$= (6h + h_p)^{1/2}$
β	$= (2h + h_p)^{1/2}$

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I. Introduction

THE control of large dynamic motions of space structures is a subject of considerable importance in connection with the deployment of large structures in outer space for various missions. The space structures are very flexible in most cases, and therefore necessitate the control of elastic deformations in addition to rigid motions for proper performance. Piezo-electric materials, which exhibit mechanical deformations when an electric field is applied, have recently received attention because of their potential application to the control of the flexible structure.[1,2] These materials, bonded to the surface of a structural element, transfer forces to the structural member according to the magnitude of exitation voltage applied to them. These forces exerted by the piezo-electric actuators may be employed to actively control the deformations of the structure. Recently Crawly and de Luis proposed a static model of the mechanical coupling of such "segmented piezo-actuators" bonded to a beam element with the dynamic deformation of the beam. No structural forces other than a pure bending moment upon the cross section of beam was considered in their model. Thus, the model in Ref. 2 does not account for the effects of the transverse shear and axial forces in the beam on the magnitudes of shear stresses τ' and τ'', which are exerted on the beam by the piezo-actuator. The purpose of the present work is to propose a refined model which takes into account the axial force as well as the transverse shear force in the structural member (beam column) in formulating the governing equations for the shear stresses transferred to the structural member by a piezo-electric actuator bonded to the structure.

These control forces can be included as external forces acting on the space-truss/frame, in the nonlinear dynamic analysis models developed by Kondoh and Atluri,[3,5] Tanka, et al.[4] and Shi and Atluri.[6,7] In these works, explicit expressions for the tangent stiffness matrices of each beam column in a three-dimensional lattice structure undergoing large deformations, incorporating exactly the effects of nonlinear bending-stretching coupling, have been derived. In as much as the control forces exerted by the piezo-actuators are functions of the applied voltages in each actuator segment, the "equivalent nodal force vector" would also be a function of the applied voltages.

It is the object of the control algorithm, then, to determine the appropriate voltages to be applied to the actuators in order to damp out the nonlinear dynamic deformations of the lattice-type structure.

In Sec. II, we first consider the exact equilibrium equation for a segment of a beam column to the upper and lower surfaces of which, piezo-electric liners are bonded. We neglect the inertia forces and use a static analysis in formulating the equilibrium equations of this beam-column segment. Under the assumption that the piezo-electric liner is very thin, we manipulate the integral form of equilibrium equations together with the compatability relations and the stress-strain relations in order to obtain a differential form of governing equations for the forces and moments acting on the cross section of the beam column and the shear stresses τ' and τ'' transferred to it by the piezo-actuator. These governing equations, together with appropriate end conditions, will determine the desired expression for the shear stresses transferred to the beam column in terms of the geometry of the beam column and the end forces and moments.

In Sec. III, we consider segmented piezo-actuators which are distributed along the length of the beam column to obtain the expression for the shear stresses τ' and τ''. The beam-column segment, formed by cutting at both ends of the piezo-actuator, is assumed to be sufficiently short so that its buckling load is very large compared with the axial force applied upon it. It is seen that the end conditions of Crawly and de Luis,[2] in which they prescribed two different bending strains at the ends of the beam segment, are not compatible with the moment equilibrium of their model. The result of the present work, as opposed to that of Crawly and de Luis,[2] accurately reflects the effect of the axial force as well as the transverse shear force upon the shear stresses τ' and τ'' transmitted by a segmented piezo-actuator. The influences of these forces are demonstrated through numerical examples, and this is followed by some discussion of the numerical results. A brief synopsis of the present work, which was completed in 1987, has been included in the survey article by Atluri and Iura.[8]

II. Governing Equations of a Beam Column Lined with Piezo-Actuators and Undergoing Large Deflections

A piezo-electric material, bonded to the upper and lower surfaces of a beam column, transfers forces to the structural element according to the magnitude of exitation voltage applied to it. Such actuator force fields may be used to control the dynamic deformations of a space frame, each member of which is modeled as a beam column. For such control applications, the expressions of the forces transferred to the structural element are required in terms of the excitation voltage. In this section, we examine the deformations of a beam column, lined with piezo-actuators on its upper and lower surfaces, to establish the governing equations for the forces and moments transferred to the beam column. The assumptions of small strains and linear elastic material behavior are made, but we do not place any restrictions upon the magnitude of deflections and rotations in formulating the governing equations. We restrict our attention in formulating the governing equations in this paper only to the case of planar motions of the beam column.

Consider a segment of the beam column of length L, along which the piezo-electric actuator is lined (see Figs. 1, 2). The thickness and length of the upper and lower piezo-actuators are assumed to be the same, and we take the width of the beam column and the actuator to be unified for convenience. The piezo-actuators are assumed to be bonded to the surfaces of the beam column by very thin layers of adhesive.

The kinematic assumptions in the present analysis are summarized as follows:

a) The length of the piezo-actuator segment L (see Fig. 1) is assumed to be much smaller than the total length of the beam column itself. Thus, along a single beam column there may be several piezo-actuators.

b) The total beam column may undergo large deformations with arbitrarily large rotations but small strains.

c) The segment of the beam column along which a piezo-actuator is bonded may also undergo large rotations from the undeformed configuration. If, in the deformed configuration, the ends of the beam-column segment (along which a piezo-actuator is bonded) are joined by a straight line, it is assumed that a) this straight line may be oriented at an arbitrary angle to the undeformed axes of the beam; however b) the local elastic rotations of the differential elements of the beam with respect to this straight line, in the deformed configuration are assumed to be small.

In the undeformed configuration, we employ a rectangular Cartesian coordinate system with origin at the left end and take the x-axis to be along the line of centroids and the z-axis to be downward. In the deformed configuration, a rigid rotation is imposed upon the deformed segment of the beam column so that the line connecting the two centroids at both ends is considered to be horizontal (see Fig. 2). Then we take the y_1-axis to be along that horizontal line and the y_3-axis to be downward.

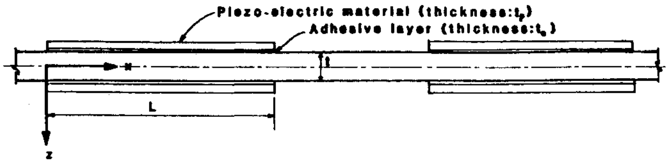

Fig. 1 Beam column bonded with a piezo-electric material.

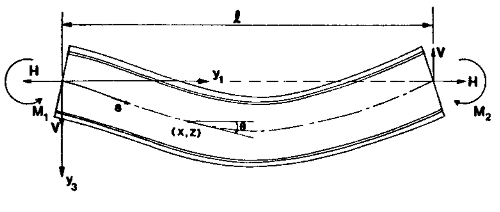

Fig. 2. Deformed beam-column segment.

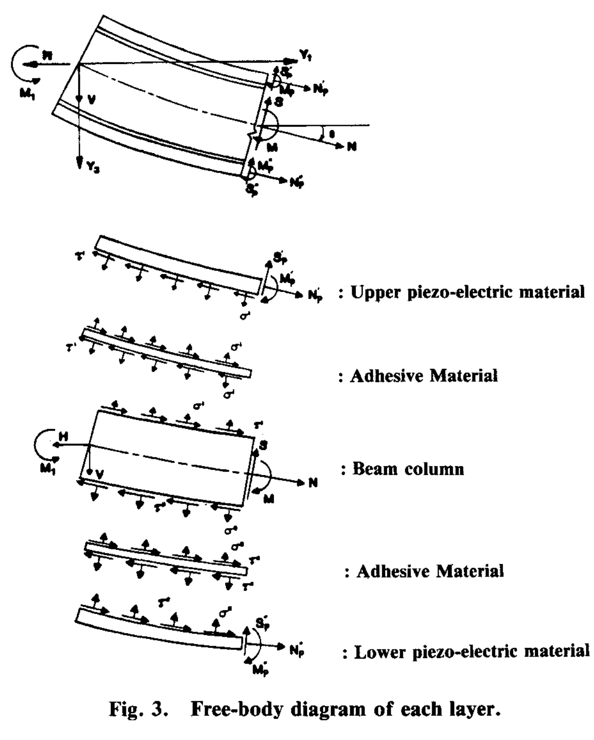

Fig. 3. Free-body diagram of each layer.

We denote by θ the angle between the tangent to the deformed centroidal axis and the y_1 axis. The coordinates of the deformed centroidal axis of the beam column in the y_1-y_3 coordinate frame are denoted by (X, Z). We then have

$$\frac{dX}{ds} = \frac{1}{(1+e)}\frac{dX}{dx} = \cos\theta, \qquad \frac{dZ}{ds} = \frac{1}{(1+e)}\frac{dZ}{dx} = \sin\theta \quad (1)$$

where s is a coordinate along the deformed centroidal axis and e is the extensional strain along the centroidal axis.

We first consider the balance equations for forces and moments for each slice of the beam column between $s = 0$ and an arbitrary value of s. After differentiation and appropriate manipulation of these balance equations, we can obtain the differential form of equilibrium equations. From Fig. 3, we have the following.

A. Upper Piezo-actuator

$$N_p' \cos\theta + S_p' \sin\theta - \int_0^s \tau' \cos\bar{\theta}\, d\bar{s} - \int_0^s \sigma' \sin\bar{\theta}\, d\bar{s} = 0 \quad (2)$$

$$-N_p' \sin\theta + S_p' \cos\theta + \int_0^s \tau' \sin\bar{\theta}\, d\bar{s} - \int_0^s \sigma' \cos\bar{\theta}\, d\bar{s} = 0 \quad (3)$$

$$\begin{aligned} M_p' &- \int_0^s \tau' \cos\bar{\theta}\left(Z - \bar{Z} - \frac{t_p}{2}\cos\bar{\theta}\right)d\bar{s} \\ &+ \int_0^s \tau' \sin\bar{\theta}\left(X - \bar{X} - \frac{t_p}{2}\sin\bar{\theta}\right)d\bar{s} \\ &- \int_0^s \sigma' \cos\bar{\theta}\left(X - \bar{X} - \frac{t_p}{2}\sin\bar{\theta}\right)d\bar{s} \\ &- \int_0^s \sigma' \sin\bar{\theta}\left(Z - \bar{Z} - \frac{t_p}{2}\cos\bar{\theta}\right)d\bar{s} \end{aligned} \quad (4)$$

B. Lower Piezo-actuator

$$N_p'' \cos\theta + S_p'' \sin\theta + \int_0^s \tau'' \cos\bar{\theta}\, d\bar{s} + \int_0^s \sigma'' \sin\bar{\theta}\, d\bar{s} = 0 \quad (5)$$

$$-N_p'' \sin\theta + S_p'' \cos\theta - \int_0^s \tau'' \sin\bar{\theta}\, d\bar{s} + \int_0^s \sigma'' \cos\bar{\theta}\, d\bar{s} = 0 \quad (6)$$

$$\begin{aligned} M_p'' &+ \int_0^s \tau'' \cos\bar{\theta}\left(Z - \bar{Z} + \frac{t_p}{2}\cos\bar{\theta}\right)d\bar{s} \\ &- \int_0^s \tau'' \sin\bar{\theta}\left(X - \bar{X} - \frac{t_p}{2}\cos\bar{\theta}\right)d\bar{s} \\ &+ \int_0^s \sigma'' \cos\bar{\theta}\left(X - \bar{X} - \frac{t_p}{2}\sin\bar{\theta}\right)d\bar{s} \\ &+ \int_0^s \sigma'' \sin\bar{\theta}\left(Z - Z + \frac{t_p}{2}\cos\bar{\theta}\right)d\bar{s} = 0 \end{aligned} \quad (7)$$

C. Beam column

$$\begin{aligned} \mathrm{N}\cos\theta - H + S\sin\theta &+ \int_0^s \tau' \cos\bar{\theta}\, d\bar{s} - \int_0^s \tau'' \cos\bar{\theta}\, d\bar{s} \\ &+ \int_0^s \sigma' \sin\bar{\theta}\, d\bar{s} - \int_0^s \sigma'' \sin\bar{\theta}\, d\bar{s} = 0 \end{aligned} \quad (8)$$

$$\begin{aligned} -N\sin\theta - V + S\cos\theta &- \int_0^s \tau' \sin\bar{\theta}\, d\bar{s} \\ &+ \int_0^s \tau'' \sin\bar{\theta}\, d\bar{s} + \int_0^s \sigma' \cos\bar{\theta}\, d\bar{s} - \int_0^s \sigma'' \cos\bar{\theta}\, d\bar{s} = 0 \end{aligned} \quad (9)$$

and

$$\begin{aligned} M - M_1 &- XS\cos\theta - ZS\sin\theta - ZN\cos\theta + XN\sin\theta \\ &+ \int_0^s \tau' \cos\bar{\theta}\left(\frac{t}{2}\cos\bar{\theta} - \bar{Z}\right)d\bar{s} \\ &+ \int_0^s \tau' \sin\bar{\theta}\left(\frac{t}{2}\sin\bar{\theta} + \bar{X}\right)d\bar{s} \\ &+ \int_0^s \tau'' \cos\bar{\theta}\left(\frac{t}{2}\cos\bar{\theta} + \bar{Z}\right)d\bar{s} \\ &- \int_0^s \tau'' \sin\bar{\theta}\left(\bar{X} - \frac{\mathrm{X}}{2}\sin\bar{\theta}\right)d\bar{s} - \int_0^s (\sigma' - \sigma'')\cos\bar{\theta}\bar{X}ds \\ &- \int_0^s (\sigma' - \sigma'')\sin\bar{\theta}\bar{Z}d\bar{s} = 0 \end{aligned} \quad (10)$$

Here a superposed bar " – " indicates a dummy variable, for example, $\bar{\theta} = \theta(\bar{s})$.

Differentiating Eqs. (2) through (10) with respect to the length variable s, and using Eq. (1) while invoking the assumption that θ is small (which renders $\cos\theta \approx 1$ and $\sin\theta \approx \theta$), we have

$$\frac{dN_p'}{ds} - \tau' + S_p'\frac{d\theta}{ds} = 0, \qquad \frac{dS_p'}{ds} - \tau' - N_p'\frac{d\theta}{ds} = 0 \quad (11a)$$

$$\frac{dM_p'}{ds} - S_p' \frac{\tau' t_p}{2} = 0 \quad (11b)$$

$$\frac{dN_p''}{ds} + \tau'' + S_p''\frac{d\theta}{ds} = 0, \qquad \frac{dS_p''}{ds} + \sigma'' - N_p''\frac{d\theta}{ds} = 0 \quad (12a)$$

$$\frac{dM_p''}{ds} - S_p'' + \frac{\tau'' t_p}{2} = 0 \quad (12b)$$

$$\frac{dN}{ds} + \left(\tau' - \tau''\right) + S\frac{d\theta}{ds} = 0, \qquad \frac{dS}{ds} + \left(\sigma' - \sigma''\right) - N\frac{d\theta}{ds} = 0, \quad (13a)$$

$$\frac{dM}{ds} - S + \frac{t}{2}\left(\tau' + \tau''\right) = 0 \quad (13b)$$

Since the extensional strain of the centroidal axis is assumed to be small, for our present purpose, we can replace (ds) by (dx) in the preceding equations.

From the overall equilibrium (See Fig. 2 and Fig. 3), we obtain the following relations

$$V = (M_2 - M_1)/l \quad (14a)$$

$$S = V\cos\theta + H\sin\theta - (S_p' + S_p'') \quad (14b)$$

$$N = H\cos\theta - V\sin\theta - (N_p' + N_p'') \quad (14c)$$

$$\begin{aligned} M = M_1 + HZ + VX &+ \frac{t + t_p}{2}(N_p'' - N_p'') \\ &- M_p'' - M_p' \end{aligned} \quad (14d)$$

The above equilibrium equations, except Eq. (14d), are completely compatible with the differential forms of equilibrium Eq. (11) through (13). Equation (14d) has a minor discrepancy with these equations when S_p' and S_p'' are not equal to each other. In most cases, the piezo-actuators are sufficiently thin $(t_p/t \ll 1)$ so that we can neglect the normal stress components (σ', σ''), the transverse shear forces (S_p', S_p''), and the moments (M_p', M_p''). Under these conditions the moment balance equations, Eqs. (11b) and (12b) can be neglected, and we obtain the following simplified equations

$$\frac{dN_p'}{dx} - \tau' = 0, \qquad \frac{dN_p''}{ds} + \tau'' = 0 \tag{15}$$

$$\frac{dN}{dx} + \left(\tau' - \tau''\right) + S\frac{dQ}{dx} = 0, \tag{16a}$$

$$\frac{dS}{dx} - N\frac{d\theta}{dx} = 0 \tag{16b}$$

$$\frac{dM}{dx} - S + \frac{t}{2}\left(\tau' + \tau''\right) = 0 \tag{16c}$$

In the above five equations, we have eight unknowns N_p', N_p'', τ', τ'', N, S, M, θ, and, thus, three more equations are required to form a complete set of governing equations. These three equations are obtained from the stress-strain relations and the compatibility equation. The moment curvature and the axial force-extension relations are given by

$$M = \frac{E}{12}t^3\frac{d\theta}{dx} \tag{17}$$

and

$$N = E \cdot (\mathrm{et}) \tag{18}$$

Assuming that the adhesive layers are perfectly bonded to the surfaces of the piezo-actuator and the beam column, we write the transferred shear stresses as

$$\tau' = G_a\gamma' = G_a(u_p' - u_f')/t_a$$
$$\tau'' = G_a\gamma'' = G_a(u_f'' - u_p'')/t_a \tag{19}$$

where u_p' and u_p'' are the tangential displacement of the piezo-actuator, u_f' and u_f'' are the tangential displacements at the upper and lower fibers of the beam-column. The compatibility relations at the upper and lower surfaces of the beam-column can be written as

$$\frac{du_f'}{dx} = e + \frac{t}{2}\frac{d\theta}{dx} \qquad \frac{du_f''}{dx} = e - \frac{t}{2}\frac{d\theta}{dx} \tag{20}$$

Denoting by V' and V'', the excitation voltages applied to the upper and lower piezo-actuators, we write the axial force-extension relations for the piezo-electric material as

$$N_p' = E_pt_p\left(\frac{du_p'}{dx} - \frac{cV'}{t_p}\right) = E_pt_p\left(\frac{du_p'}{dx} - \Lambda'\right) \tag{21}$$

and

$$N_p'' = E_pt_p\left(\frac{du_p''}{dx} - \frac{cV''}{t_p}\right) = E_pt_p\left(\frac{du_p''}{dx} - \Lambda''\right) \tag{22}$$

where c is the piezo-electric constant relating the voltages to the mechanical strains Λ' and Λ'' induced by the voltages.

Now manipulating the preceding Eqs. (17)–(22), we can eliminate the displacements u_p', u_p'', u_f', u_f'', and obtain the two equations

$$\frac{d\tau'}{dx} = \frac{G_a}{t_a}\left(\frac{N_p'}{E_pt_p} + \Lambda' - \frac{N}{Et} - \frac{6M}{Et^2}\right) \tag{23a}$$

$$\frac{d\tau''}{dx} = \frac{G_a}{t_a}\left(-\frac{N_p''}{E_pt_p} - \Lambda'' + \frac{N}{Et} - \frac{6M}{Et^2}\right) \tag{23b}$$

The complete set of eight governing equations are now given by Eqs. (15), (16), (17), and (23). As seen from these equations, the shear stresses τ' and τ'', transmitted by the piezo-actuators to the beam column, are dependent upon the force (N) and moment (M) as well as the mechanical strains Λ' and Λ'' induced by the excitation voltages.

The boundary conditions for the aforementioned governing equations are given as

$$N_p' = 0 \qquad N_p'' = 0, \qquad \text{at } x = 0, L \tag{24}$$

$$M = M_1 \quad \text{at } x = 0, \qquad M = M_2 \quad \text{at } x = L \tag{25}$$

$$\theta = \theta_1 \quad \text{at } x = 0, \qquad \theta = \theta_2, \quad \text{at } x = L \tag{26}$$

Thus, we have established the complete set of governing equations and the associated boundary conditions.

III. Forces Transmitted to the Beam Column by the Piezo-actuator

The shear stresses τ' and τ'', transmitted by the piezo-actuator to the beam column, depend upon the excitation voltages applied to the actuators as well as on the forces and moment acting upon the cross section of the beam column. These actuator forces are used to control the overall deformations of the structure. The axial force (N) and the bending moment (M) in each beam column are quantities that depend upon the external loading on the structure. In order to predict the response of the structure for a given set of excitation voltages, we need to express the shear stresses τ' and τ'' in terms of the strains Λ' and Λ'' induced by the applied voltages and the loading parameters, N and M.

For the purpose of control, it is desirable to have segments of actuators distributed along the structure because this enables us to vary the input exitation voltage along the length of the structure.[2] Moreover, most of the transfer of actuator shear forces to the beam column occurs near the ends of the actuator segment.[2] It is therefore more effective to have many short segments of actuators, rather than one long piezo-actuator. Thus, we assume that the length of the piezo-actuator (and thus the beam column segment which is lined with the piezo-actuator) is not long so that the following approximations can be made

$$\cos\theta \approx 1, \qquad \sin\theta \approx \theta, \qquad l \approx L \tag{27}$$

In order to obtain the governing equation for θ, we first manipulate Eqs. (15) and (23) to obtain

$$\frac{d^2(\tau' + \tau'')}{dx^2} - \frac{G_a}{t_at_pE_p}\left(\tau' + \tau''\right) = -\frac{12G_a}{Et^2t_a}\frac{dM}{dx} \tag{28}$$

Substituting Eq. (16c) into the above equation and using Eqs. (27) and (17), we obtain the following nondimensional equation.

$$\frac{d^4\theta}{d\bar{x}^4} - L^2\left(\frac{6G_a}{t_atE} + \frac{G_a}{t_at_pE_p} + \frac{12H}{Et^3}\right)\frac{d^2\theta}{d\bar{x}^2} + \frac{12HG_aL^4}{t_at_pE_pEt^3}\theta$$
$$= -\frac{12G_a(M_2 - M_1)L^3}{t_at_pt^3E_pE} \tag{29}$$

where $\bar{x} = x/L$ and higher order nonlinear terms such as $(d\theta/dx)^2$; $\theta\,(d^2\theta/dx^2)$; and $\theta\,(d\theta/dx^2)$ have been neglected consistent with the approximation of Eq. (27).

The term $12H/Et^3$ is negligible compared with the other two terms multiplied by $d^2\theta/dx^2$ because $|H/t| \ll G_at/t_a$, and $|H/t| \ll G_a(t/t_a)(t/t_p)\,(E/E_p)$. The last term on the left side of Eq. 29 represents the contribution of the horizontal axial force to the flexural deformation. It is negligible if θ is very small and if the axial force is small compared with the buckling load of the beam-column segment. This will be the case because the beam-column segment is short; even when H is close to the critical load of the beam column itself, it will be much smaller than the critical load of the beam-column segment, which is very short compared with the whole beam column. With this approximation, we can thus obtain accurate solutions in most of the practically important cases.

Introducing the nondimensional variables

$$m = LM/EI = \frac{d\theta}{d\bar{x}}, \qquad h = \frac{G_a L^2}{t_a t E}, \qquad h_p = \frac{G_a L^2}{t_a t_p E_p} \tag{30}$$

The nondimensional parameter h characterizes the ratio of the shear rigidity of the adhesive material to the stiffness of beam; whereas the parameter h_p characterizes the ratio of the shear rigidity of the adhesive material to the stiffness of the piezo-actuator material. We rewrite the governing Eq. (29) with the aforementioned terms being neglected as

$$\frac{d^3 m}{d\bar{x}^3} - \left(6h + h_p\right)\frac{dm}{d\bar{x}} = -h_p\left(m_2 - m_1\right) \tag{31}$$

We consider the boundary conditions for the above governing equations. Two conditions are provided by Eq. (25), which are rewritten using the nondimensional variables, as

$$m = m_1 \quad \text{at } \bar{x} = 0 \quad m = m_2 \quad \text{at } \bar{x} = 1 \tag{32}$$

Another condition is obtained from Eq. (24), which is written as

$$N_p' - N_p'' = 0 \qquad \text{at } \bar{x} = 0 \qquad \text{and } \bar{x} = 1 \tag{33a}$$

$$N_p' + N_p'' = 0 \qquad \text{at } \bar{x} = 0 \qquad \text{and } \bar{x} = 1 \tag{33b}$$

Adding Eqs. (23a and b) and using Eqs. (33) and (16b) and (16c), we obtain

$$\frac{d^2 m}{d\bar{x}^2} - 6hm = -6h\xi\left(\Lambda' - \Lambda''\right) \quad \text{at } \bar{x} = 0 \quad \text{and } \bar{x} = 1 \tag{34}$$

where $\xi = L/t$. Consistent with the approximation taken in Eq. (31), we have neglected a term involving the horizontal axial force H. As will be shown later, only three of the four boundary conditions of Eqs. (32) and (34) are independent of one another. The general solution to Eq. (31) is given as

$$m = c_1 \cosh\left(\alpha\bar{x}\right) + c_2 \sinh\left(\alpha\bar{x}\right) + c_3 + \frac{h_p(m_2 - m_1)}{a_2}\bar{x}_1 \tag{35}$$

where $\alpha = (6h + h_p)^{1/2}$.

Applying the boundary conditions of Eqs. (32) and (34), we determine the constants c_1, c_2, c_3 as

$$c_1 = 6h[m_1 - \xi(\Lambda' - \Lambda'')]/\alpha^2 \tag{36a}$$

$$c_2 = 6h[(m_2 - m_1 \cosh\alpha) - \xi(\Lambda' - \Lambda'')(1 - \cosh\alpha)]/\alpha^2 \sinh\alpha \tag{36b}$$

$$c_3 = [h_p m_1 + 6h\xi(\Lambda' - \Lambda'')]/\alpha^2 \tag{36c}$$

As a consequence of the assumption that the beam-column segment lined with a piezo-actuator is short, the flexural deformation is decoupled from the axial deformation, and the axial force does not appear in the above solution for the flexural deformation. The effect of the axial force upon the flexural deformation is not negligible when the magnitude of the axial force is of the order of the critical load of the beam-column segment. However, we exclude such an extreme case from the present consideration.

As solutions for the shear stresses transmitted to the beam-column, we obtain the expressions for p^+ and p^-, which are defined as

$$p+ = (\tau' + \tau'')/E_p, \qquad p- = (\tau' - \tau'')/E_p \tag{37}$$

With the aid of Eqs. (16b) and (35), we then obtain

$$p^+ = \eta\bar{G}\left[\frac{m_2 - m_1}{\alpha^2} - \frac{\{m_1 - \xi(\Lambda' - \Lambda'')\}\sinh\alpha\bar{x}}{\alpha} - \frac{\{m_2 - m_1\cosh\alpha - \xi(\Lambda' - \Lambda'')(1 - \cosh\alpha)\}\cosh\alpha\bar{x}}{\alpha\sinh\alpha}\right] \tag{38}$$

where $\eta = t/t_a$, $\bar{G} = G_a/E_p$. Noting that the last term in Eq. (16a) is negligible within the accuracy of the present model for a short beam-column segment. By combining this equation with Eqs. (15) and (23), we obtain the differential equation for $p-$, as

$$\frac{d^2 p^-}{d\bar{s}^2} - \beta^2 p^- = 0 \tag{39}$$

where $\beta = (2h + h_p)^{1/2}$. The boundary conditions for this equation are obtained by combining Eq. (23) with Eq. (33b) (which has not been used yet) as

$$\frac{dp^-}{d\bar{x}} = \eta\xi\bar{G}\left(\Lambda' - \Lambda'' - \frac{2N}{Et}\right) \approx \eta\xi\bar{G}\left(\Lambda' - \Lambda'' - 2e_0\right)$$

$$\text{at } \bar{x} = 0 \text{ and } \bar{x} = 1 \tag{40}$$

where the extension of the beam-column segment at both ends is given by $e_0 = N/Et$ within the present approximation. The conditions of Eq. (40) are associated with the axial deformation of the beam-column segment, which is decoupled with the flexural deformations when the axial force is very small compared with the critical load of the beam-column segment. The axial strain may have the same order of magnitude as the strains induced in the piezo-actuator, Λ' and Λ'', but N is small compared with the critical load of the beam-column segment, which is assumed to be short.

The solution to Eq. (39) under the conditions of Eq. (40) is obtained as

$$p- = \frac{\xi\eta\bar{G}(\Lambda' + \Lambda'' - 2e_0)}{\beta}\left\{\frac{1 - \cosh B}{\sinh\beta}\cosh\beta\bar{x} + \sinh\beta\bar{x}\right\} \tag{41}$$

In order to compare with the solution of Crawley and de Luis,[2] we assume $\Lambda' = -\Lambda''$ and the axial forces at both ends are zero. Then $p^- = 0$ and $\tau'/E_p = \tau''/E_p = p^+/2$. After accounting for the difference between the coordinate system in the present study and that in Crawley and de Luis,[2] we find that the expressions for the shear stress agree with each other only for the case of pure bending (also see Fig. 4). This is due to the fact that the effect of the transverse shear force has been taken into account in the present mode; whereas such an effect was neglected in Crawley and de Luis.[2] Different bending strains at the ends of the beam segments are apparently not allowed in their model, and, therefore, the boundary conditions in which they prescribed two different strain values at the ends of the segment are not compatible with the overall moment equilibrium of their model. To illustrate the shear transmitted by the piezo-actuators, we consider the following material and geometrical data and the boundary condition at $x = 0$.

$$\xi = \frac{L}{t} = 10, \qquad \eta = \frac{t}{t_a} = 40$$

$$\bar{G} = \frac{Ga}{E_p} = 1/63, \quad h = G_a L^2 / t_a t E = 57, \quad h_p = \frac{G_a L^2}{t_a t_p E_p} = 423$$

$$\Lambda' = -\Lambda'' = 10^{-3}, \qquad m_1 = 10^{-3}$$

The above material data corresponds approximately to that of an aluminum beam column, epoxy adhesive, and ceramic piezo-actuator. We first plot the shear-stress distribution in Fig. 4 when there is no axial force in the beam column. We have a similar trend for other values of transverse shear force

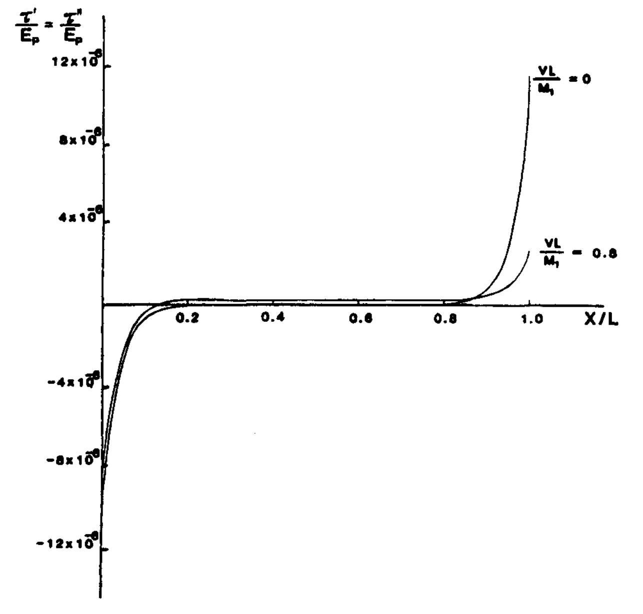

Fig. 4 Effect of shear force in the beam upon the distribution of shear stress exerted by the piezo-actuator when there is not axial force.

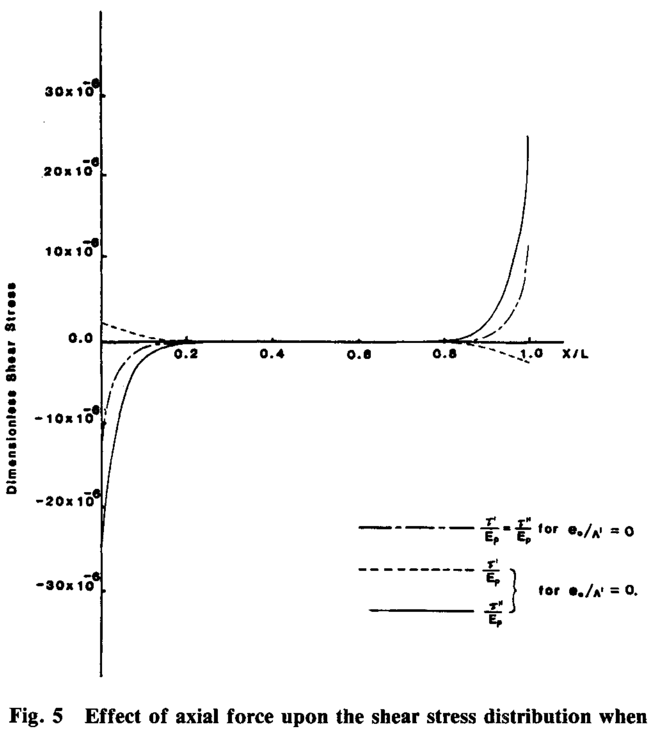

Fig. 5 Effect of axial force upon the shear stress distribution when there is not sheer force.

in the beam column, and, therefore, only one case with a nonzero transverse shear force is shown. When the transverse shear force in the beam is zero, i.e., $(VL/M_1) = 0$, the present results agree with those of Crawley and de Luis.[2] On the other hand, a nonzero transverse shear force (see the case of $VL/M_1 = 0.8$ in Fig. 4) in the beam contributes to a significant change in the distribution of the shear stress exerted by the actuator as shown in Fig. 4. Also, as seen from Fig. 4, the degree of localization in the transmitted shear stress at the two ends of the actuator segment may be very different depending on the magnitude of the transverse shear force in the beam. It should be noted, on the other hand, that the Crawley-de Luis[2] solution is not applicable in the case when $(VL/M_1) \neq 0$.

Because of the assumption that the piezo-actuator segment is short, the flexural deformation of the piezo-actuator segment is decoupled with its axial deformation, and the effect of the axial force in the beam upon its flexural deformation is negligible. However, the axial force in the beam has a significant effect on the shear stress transmitted by the piezo-actuator to the beam column because the axial force transferred to the piezo-actuator changes the response of the piezo-actuator by inducing deformation in it. For numerical illustration, the distribution of the shear stress exerted by the actuator is plotted in Fig. 5, for a case when there is no transverse shear force in the beam, but there exists a nonzero axial force in the beam. Due to the axial force in the beam, the shear stress exerted by the upper actuator has a totally different distribution as compared to the shear stress exerted by the lower actuator.

As another example, we consider both the transverse shear and axial forces in the beam to be nonzero and plot the distributions of the transmitted shear stresses in Fig. 6. Compared with the preceding two cases, the distribution of the transmitted shear stress may be more complex; however, we still observe the trend of stress localization around the ends of the segment.

Finally, it is recalled that in the present study only the rotation of one end of the beam-column segment relative to the other is assumed to be small because the beam-column segment is short. Further, we imposed an appropriate rigid rotation, which can be finite, to bring the deformed beam-column segment to the configuration in Fig. 2 so that the line connecting

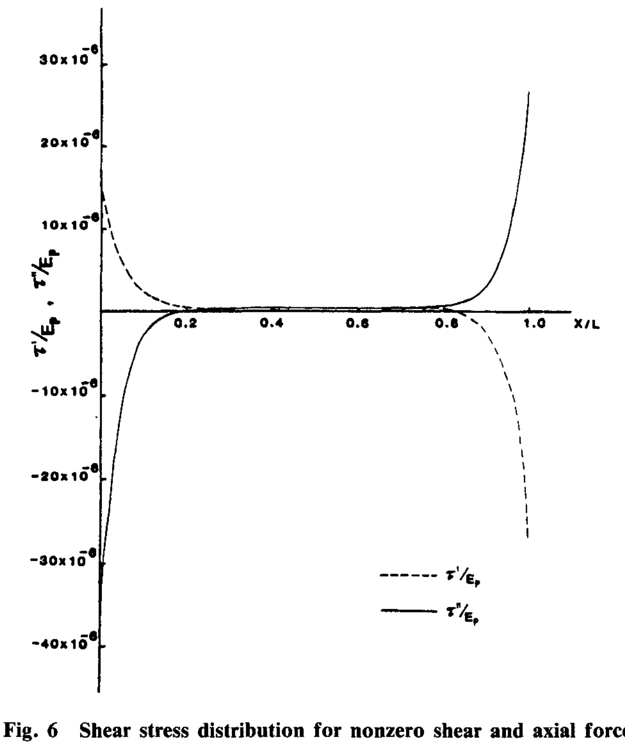

Fig. 6 Shear stress distribution for nonzero shear and axial forces $\frac{VL}{M_1} = 1 e_0/\Lambda' = 1$.

the two ends is horizontal. Therefore, the present result is applicable to the case of the slender flexible structures undergoing large deflections and rotations if it is combined with the special finite element method as given by Kondoh and Atluri.[3,5]

The use of the present analysis in implementing an active control of nonlinear dynamic response of three-dimensional, lattice-type structures, wherein each member carries axial loads, transverse shear loads, and moments, is discussed in detail in a recent report.[7]

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